

A large graphic on the right side of the image features a teal circle with a white crescent moon and a white circle inside. A dark grey circle is positioned in the center, containing a white globe icon. The word "CERAVISION" is overlaid on this graphic, with "CERAVI" in teal and "SION" in yellow.

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25 October 2023

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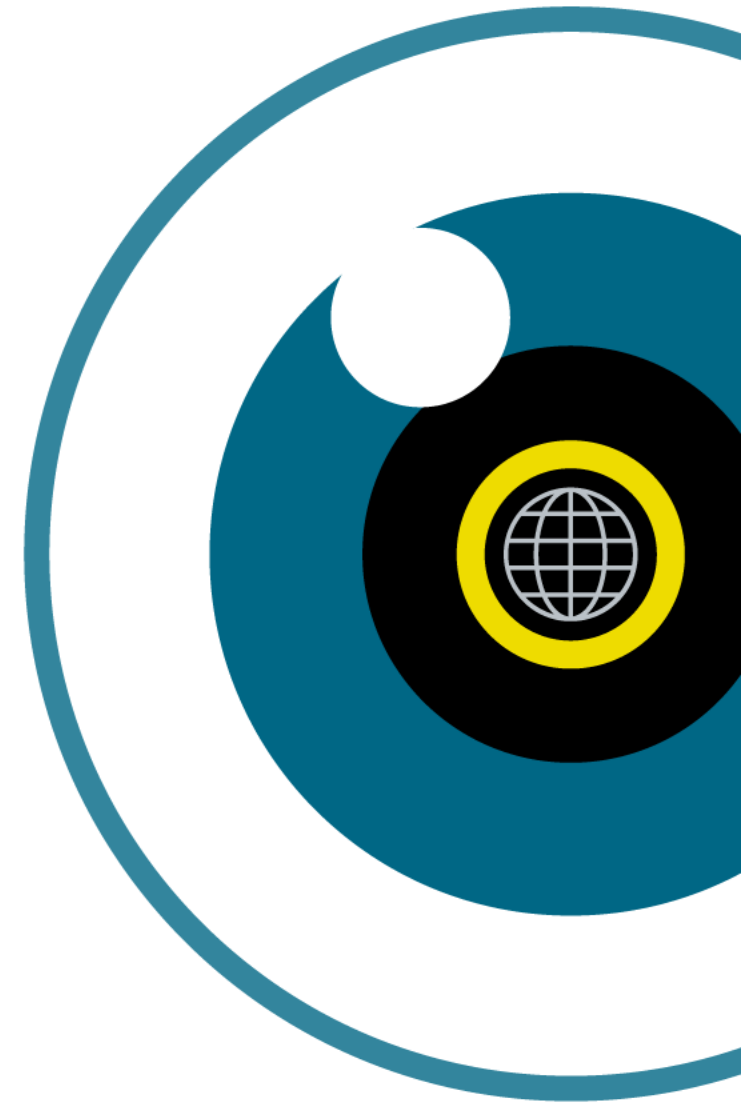


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Managing Climate Change Risks in Insurance

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Agenda of the talk

- Preliminaries drawn from IFoA and actuarial associations
- Some initial climate change impact modelling projects
- Managing risks in presence of climate change: worsening risks
- Managing risks in presence of climate change: uninsurability
- One potential useful risk control: prevention (before and after the claim)



Climate change risk taxonomy (reminder)

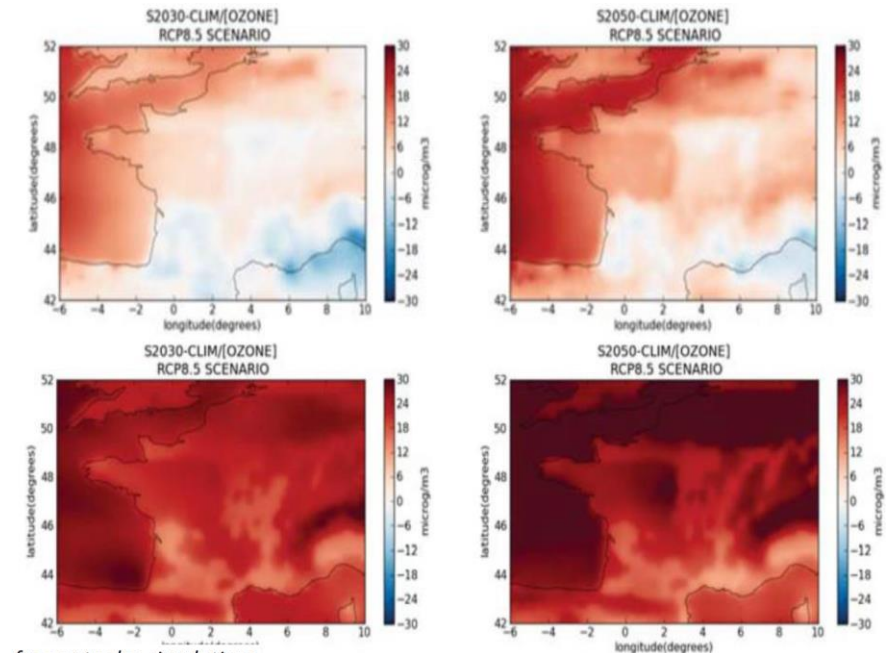
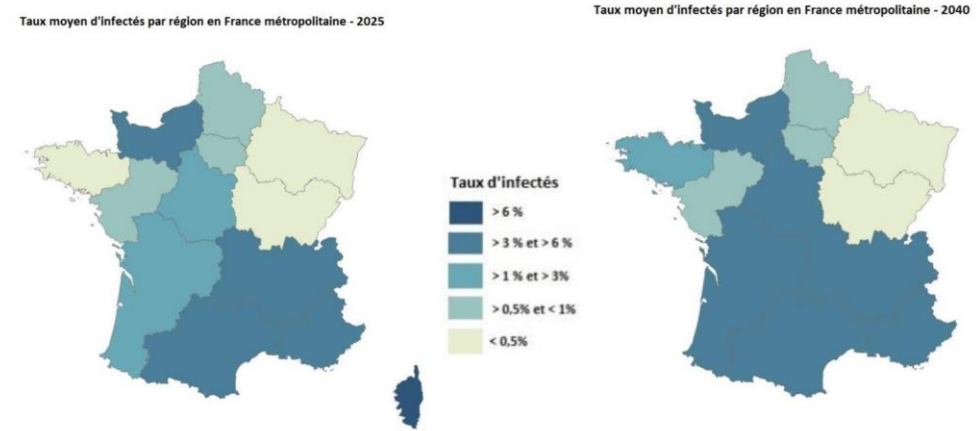
- The effects of climate change present various risks, not limited to those directly attributable to weather or temperature.
- **“Physical”** risks describe those risks emerging from climate factors, such as extreme high temperature or rising sea levels,
- **“Transition”** risks are those that emerge from a societal shift towards a low-carbon economy, and
- **“Liability”** risks are those that arise from parties who have suffered loss and damage from climate change, and seek to recover such losses from others

Climate change impacts on insurance (IFoA)

Risk Class	Physical	Transition	Liability
Market	Yes	Yes	Yes
Longevity	Yes	Less material	No
Mortality/Morbidity	Yes	Less material	No
Lapse	Less material	Yes	No
Counterparty	Yes	Yes	Yes
Operational	Less material	Yes	No
Strategic	Yes	Yes	Yes
Reputational	n/a	Yes	Yes

Modelling impact on health and mortality insurance

- Extreme heat waves
- Vectorial diseases
- Pollution
- Co-benefits
- ACPR, Drif and Valade (2020) ->



Current works in progress

- Develop an add-on of the StMoMo Package that includes refinements of Lee-Carter type mortality models taking into account temperature evolutions
- Refine stress tests proposed by ACPR and by AON Benfield (Drif and Valade 2020)
- Quickest detection of mortality level and trend changes with additional information

Modelling impact on P&C insurance

- Increase in claim frequency (sea-level rise, subsidence risk)
- Combining insurer data and external open data (with quality issues)
- Risk of de-mutualisation / exclusion
- Increase in claim severity (hurricane, ...)
- Uncertainty on claim severity and frequency
- Prevention before the event
- Prevention after the event

Subsidence risk: thesis of P. Chatelain (2023)

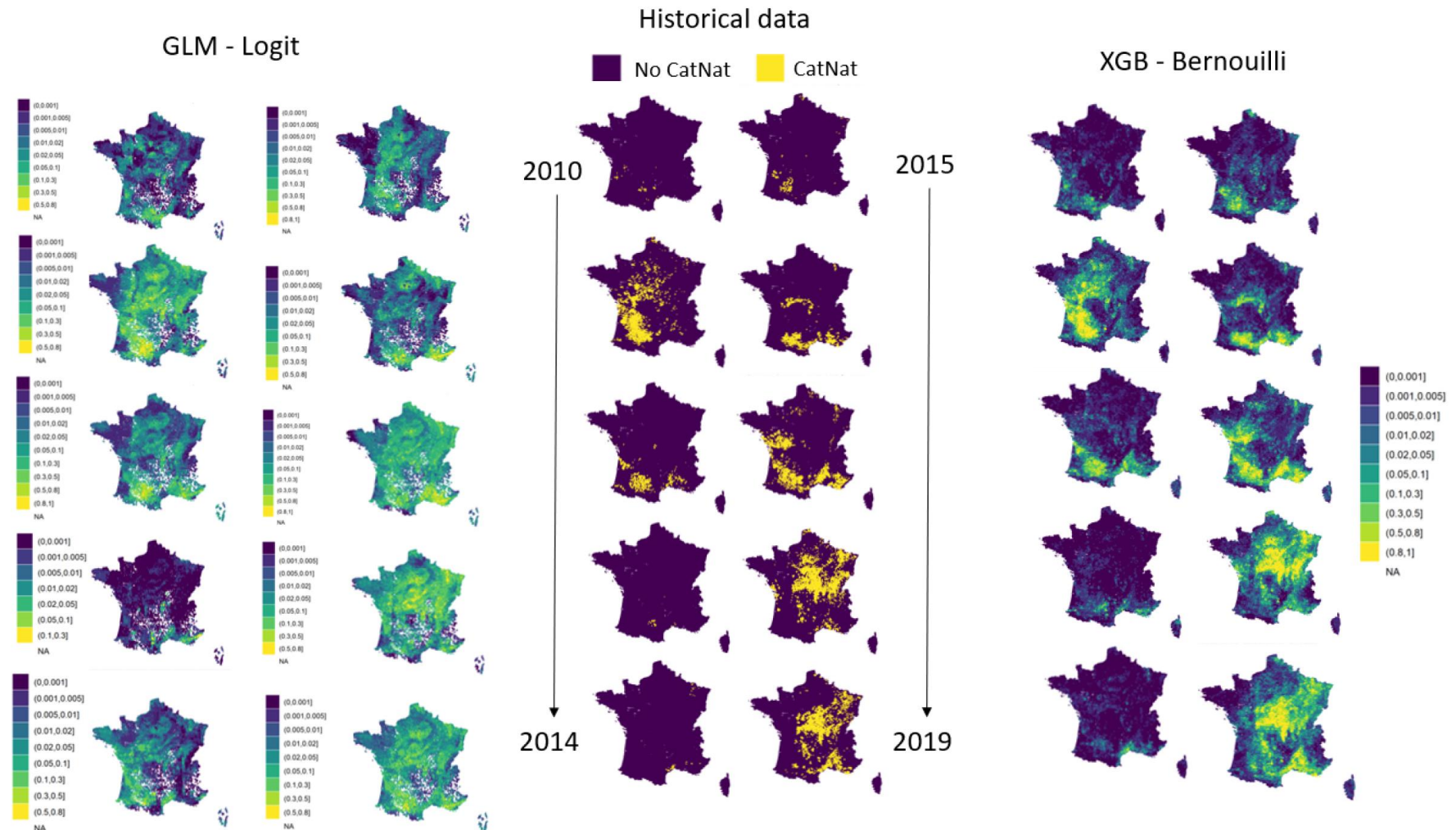
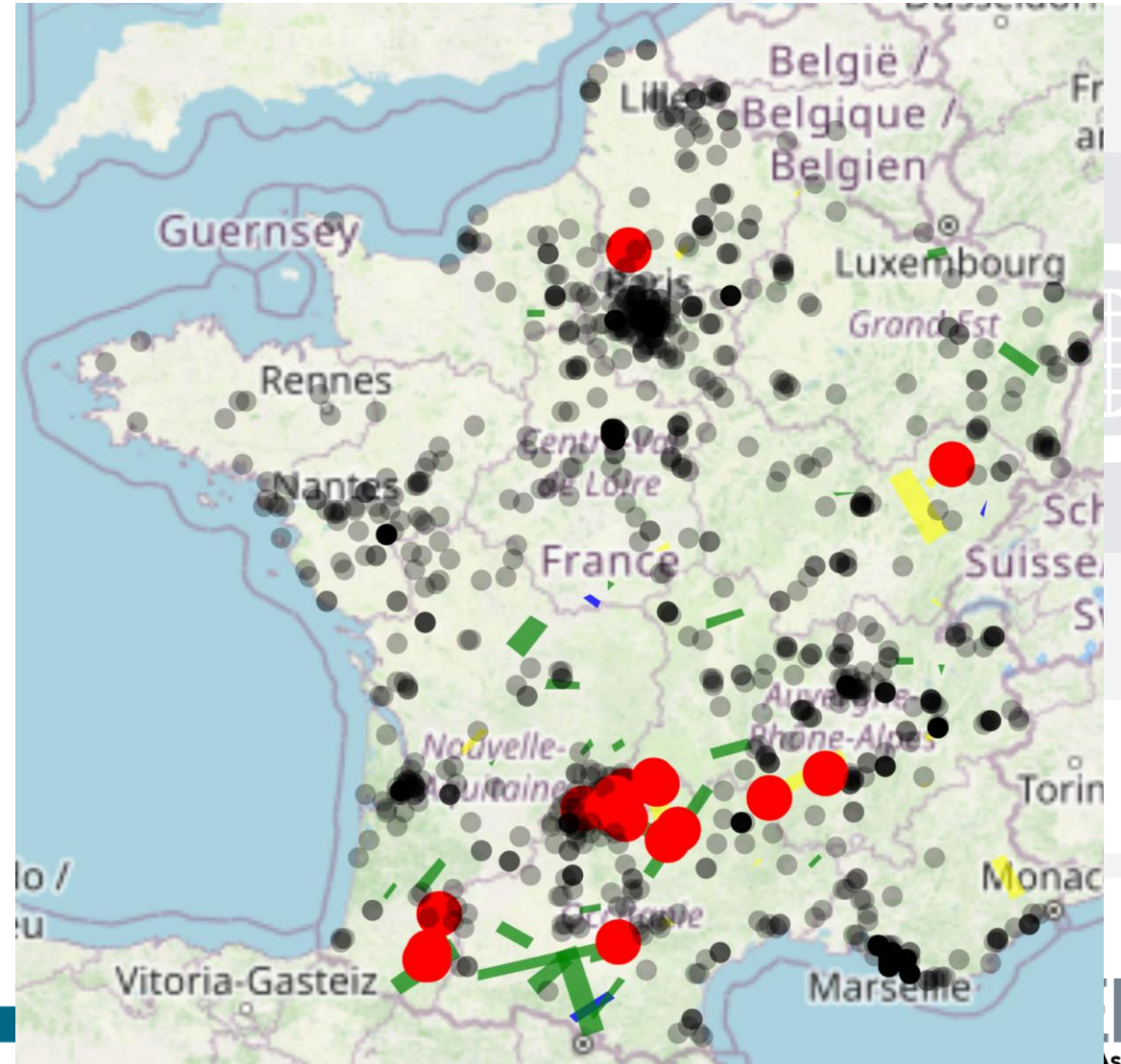
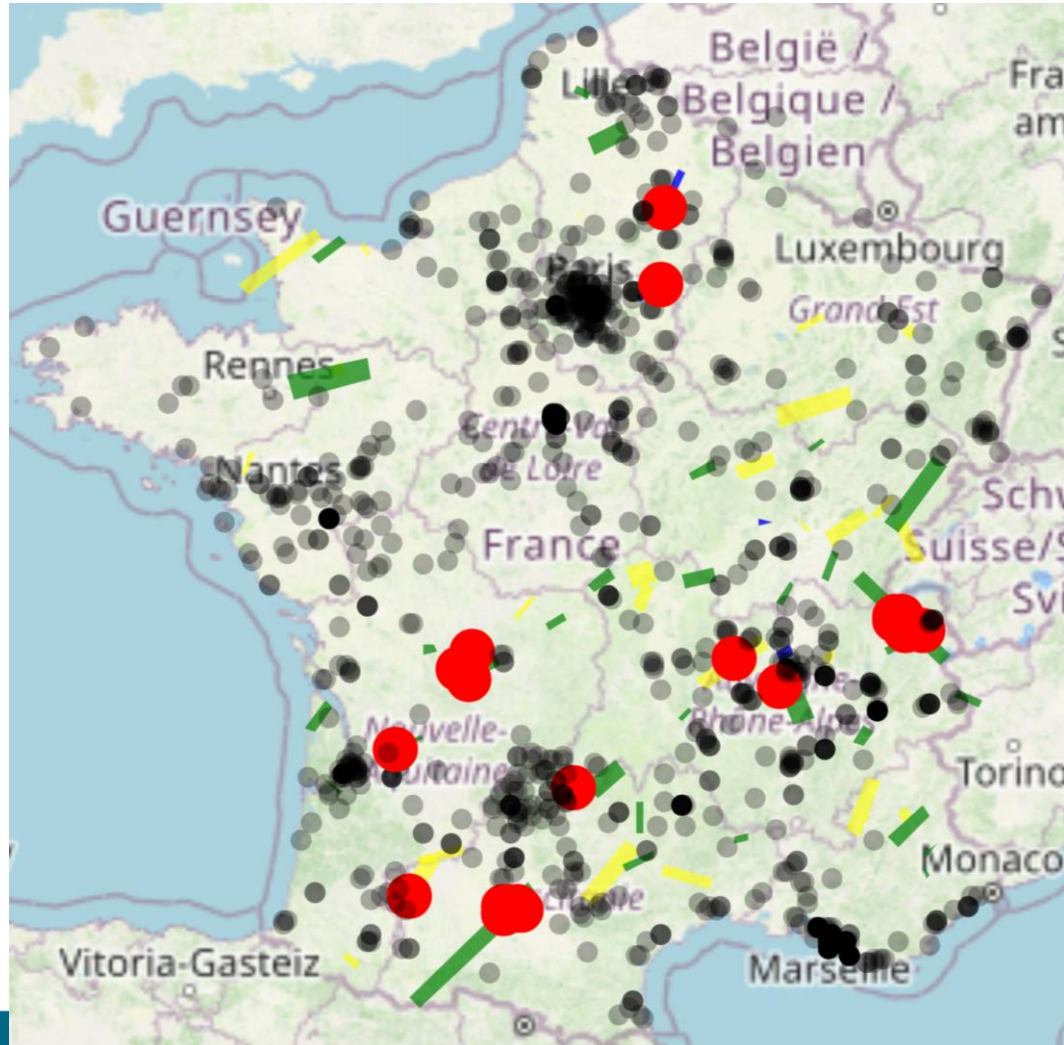


Figure 5: Comparison between the observed CatNat declaration, the GLM logit (using RDI magnitude, SSWI severity and SPI magnitude) and the XGBoost using all the variables calculated from the three indexes.

Hail risk and climate change (work in progress with Rayane Vigneron)



Risks for the insurer: bankruptcy, insolvency, or mass lapse

- Ruin may occur
- Ruin probability may become too high (insolvency)
- Risk may become uninsurable (premium too high or « infinite mean »)
- Premium may be adjusted instantaneously or not: no adjustment (uncertainty), perfect adjustment or credibility adjustments
- Previous works with Albrecher and Constantinescu, Kortschak and Ribereau
- Work in progress with Albrecher and Guerra, and with Mamode Khan and Minier

Classical ruin model

$$R(t) = u + ct - \sum_{k=1}^{N(t)} X_k,$$

- $u \geq 0$: initial surplus
- $(N(t))_{t \geq 0}$ claim counting process (Poisson or renewal process)
- X_k iid random variables: claim amounts, $E\{X\} < \infty$
- $c > 0$: constant premium intensity
- $ct > E\{N(t)\}E\{X\}$: net profit condition

Ruin

Time of ruin

$$T_u = \inf_{t \geq 0} (R(t) < 0 \mid R(0) = u)$$

Probability of ruin

$$\psi(u) = P(T_u < \infty)$$

Classical result (Cramer, 1930) for compound Poisson model with $X \sim \text{Exp}(\theta)$ claim amounts

$$\psi(u) = \min \left\{ \frac{\lambda}{\theta c} e^{-(\theta - \frac{\lambda}{c})u}, 1 \right\}, \quad u \geq 0.$$

Mixing idea

Denote by

$$\psi_{\theta}(u) = P(T_u < \infty \mid \Theta = \theta)$$

Then, the ruin probability is given by

$$\psi(u) = \int_0^{\infty} \psi_{\theta}(u) dF_{\Theta}(\theta).$$

Claims: dependent Pareto

- Compound Poisson risk model ($\tau \sim \text{Exp}(\lambda)$)
- Claims $X \sim \text{Exp}(\Theta)$, where $\Theta \sim \Gamma(\alpha, \beta)$

Ruin probability is

$$\begin{aligned} \Psi(u) &= \int_0^{\lambda/c} 1 \cdot \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\beta\theta} d\theta \\ &+ \int_{\lambda/c}^{\infty} \underbrace{\frac{\lambda}{\theta c} e^{-\theta u} e^{\frac{\lambda}{c}u}}_{\Psi_\theta(u)} \cdot \underbrace{\frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\beta\theta}}_{f_\Theta(\theta), \Theta \sim \Gamma(\alpha, \beta)} d\theta \end{aligned}$$

Claims: dependent Pareto

$$\begin{aligned}\Psi(u) &= 1 - \frac{\Gamma(\alpha, \beta\theta_0)}{\Gamma(\alpha)} \\ &+ \frac{\beta}{\Gamma(\alpha)} (\beta\theta_0)^{\alpha-1} e^{-\beta\theta_0} \underbrace{(u + \beta)^{-1}}_{\rightarrow_{u \rightarrow \infty} 0} \underbrace{\frac{\Gamma(\alpha - 1, (\beta + u)\theta_0)}{((\beta + u)\theta_0)^{\alpha-2} e^{-(\beta+u)\theta_0}}}_{\rightarrow_{u \rightarrow \infty} 1}\end{aligned}$$

One can see that the probability of ruin decays to a constant

$$\lim_{u \rightarrow \infty} \Psi(u) = 1 - \frac{\Gamma(\alpha, \frac{\beta\lambda}{c})}{\Gamma(\alpha)} > 0$$

as fast as u^{-1} !! Compared to the independent case...

Mixing dependence structure

For $X \sim \text{Exp}(\Theta)$, with $\Theta \sim F_\Theta$, consider the classical compound Poisson risk model with exponential claim sizes that fulfill, for each n ,

$$P(X_1 > x_1, \dots, X_n > x_n \mid \Theta = \theta) = \prod_{k=1}^n e^{-\theta x_k}. \quad (1)$$

That is, given $\Theta = \theta$, the X_k ($k \geq 1$) are conditionally independent and distributed as $\text{Exp}(\theta)$.

Net profit condition

Link with deceptive claim size distributions (see [24]).

Since for $\theta \leq \theta_0 = \lambda/c$ the net profit condition is violated and consequently $\psi_\theta(u) = 1$ for all $u \geq 0$, this can be rewritten as

$$\psi(u) = \int_0^\infty \psi_\theta(u) dF_\Theta(\theta) = F_\Theta(\theta_0) + \int_{\theta_0}^\infty \psi_\theta(u) dF_\Theta(\theta).$$

An immediate consequence is that in this dependence model

$$\lim_{u \rightarrow \infty} \psi(u) = F_\Theta(\theta_0),$$

which is positive whenever the random variable Θ has probability mass at or below $\theta_0 = \lambda/c$ (probability of net profit condition not being fulfilled).

Interarrival times: dependent Pareto

If one considers the classical Cramér Lundberg risk model with exponential claims, where the parameter Λ of the exponential inter-arrival time is a r.v.

$$\Lambda \sim \Gamma(\alpha, \beta)$$

one obtains Pareto inter-arrival times and the probability of ruin can be calculated explicitly as well. Similarly as before, for $\lambda_0 = \theta c$, one can write

$$\begin{aligned} \Psi(u) &= \int_0^{\lambda_0} \underbrace{\frac{\lambda}{\theta c} e^{-\theta u} e^{\frac{\lambda}{c} u}}_{\Psi_\lambda(u)} \underbrace{\frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta \lambda}}_{f_\Lambda(\lambda), \Lambda \sim \Gamma(\alpha, \beta)} d\lambda \\ &+ \int_{\lambda_0}^{\infty} 1 \cdot \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta \lambda} d\lambda \end{aligned}$$

Interarrival times: dependent Pareto

$$\Psi(u) = e^{-u\theta} \beta^\alpha \left(\beta - \frac{u}{c}\right)^{-\alpha-1} \frac{\Gamma(\alpha+1) - \Gamma(\alpha+1, c\beta\theta - u\theta)}{c\theta\Gamma(\alpha)} \\ + \frac{\Gamma(\alpha, \beta c\theta)}{\Gamma(\alpha)}$$

Thus, the probability of ruin decays again as fast as u^{-1} to

$$\lim_{u \rightarrow \infty} \Psi(u) = \frac{\Gamma(\alpha, \beta c\theta)}{\Gamma(\alpha)}$$

the probability of net profit condition not being fulfilled.

At $u = 0$

$$\Psi(0) = \frac{\Gamma(\alpha+1) - \Gamma(\alpha+1, c\beta\theta)}{\beta c\theta\Gamma(\alpha)} + \frac{\Gamma(\alpha, \beta c\theta)}{\Gamma(\alpha)}$$

Proposition

The dependence model characterized by

$$P(X_1 > x_1, \dots, X_n > x_n \mid \Theta = \theta) = \prod_{k=1}^n e^{-\theta x_k}$$

can equivalently be described by having marginal claim sizes X_1, X_2, \dots that are completely monotone, with a dependence structure due to an Archimedean survival copula with generator $\varphi = \left(\tilde{F}_\Theta\right)^{-1}$ for each subset $(X_{j_1}, \dots, X_{j_n})$ (for j_1, \dots, j_n pairwise different), where \tilde{F}_Θ denotes the Laplace-Stieltjes transform of F_Θ .

Dubey 1977

For $N(t) \sim \text{Poisson}(\Lambda)$

$$R(t) = c \int_0^t \hat{\lambda}(s) ds - \sum_{j=0}^{N(t)} Y_j, \quad t \geq 0$$

Estimates for λ

- $\hat{\lambda}(t) = E\{\Lambda \mid N(t)\} \rightarrow$ exact form for the ruin probability
- $\hat{\lambda}(t) = \frac{N(t)}{t} \rightarrow$ moments of ruin
- $\hat{\lambda}(t) = \frac{a+N(t)}{b+t} \rightarrow$ approximation of the probability of ruin, considering $\hat{\lambda}$ to be the “credibility” estimate.

Note: This model was theoretical basis for the Bonus-Malus system for the Swiss obligatory car insurance.

Asymptotic ruin probability with worsening risks, or with infinite mean risks

- Joint work with Dominik Kortschak & Pierre Ribereau (Lyon)
- Worsening risks means that the tail becomes heavier and heavier, due to phenomena like climate change or sectorial inflation.

Two models with worsening risks

Concretely let N_t be a Poisson process with intensity λ and X_t are independent Pareto distributed random variables with distribution $\bar{F}_t = (1+x/d_t)^{-\alpha_t}$ where the change of the distribution over time is characterized by

$$\mathbb{E}[X_t] = \frac{d}{\alpha_0 - 1} (1 + c_\alpha t). \quad (1)$$

We consider the risk process ($\rho > 0$)

$$R_t = u + \int_0^t (1 + \rho)\lambda \mathbb{E}[X_t] dt - \sum_{i=1}^{N_t} X_{\mathcal{T}_i} = u + \frac{(1 + \rho)\lambda d(1 + c_\alpha t)^2}{2c_\alpha(\alpha_0 - 1)} - \sum_{i=1}^{N_t} X_{\mathcal{T}_i},$$

where \mathcal{T}_i is the time of the i -th jump. Further we will denote with $S_t = \sum_{i=1}^{N_t} X_{\mathcal{T}_i}$ and

$$p(t) = \frac{(1 + \rho)\lambda d(1 + c_\alpha t)^2}{2c_\alpha(\alpha_0 - 1)}.$$

Two models with worsening risks

We will now study two sets of parameters that assure that (1) holds. In the first model we change the parameter α which means that the distribution of X_t gets more and more heavy tailed. In this case we have that $\overline{F}_t^{(1)}(x) = (1 + x/d)^{-\alpha_t}$ where

$$\alpha_t = \frac{\alpha_0 - 1}{1 + c_\alpha t} + 1.$$

we will call this variant model 1. An obvious alternative to this model 2 where we only change the other parameter. i.e. we choose $\overline{F}_t^{(2)}(x) = (1 + x/d_t)^{-\alpha_0}$ where

$$d_t = d(1 + c_\alpha t).$$

Two models with worsening risks

We now want to compare these two models. An obvious method therefore is to consider the ruin probability $\psi^{(i)}(u) = \mathbb{P}(\inf_{t>0} R_t^{(i)} \leq 0)$. Denote with $\mu = \mathbb{E}[X_0] = \frac{d}{\alpha_0 - 1}$. We get

$$\begin{aligned}\psi^{(1)}(u) &\sim \lambda \sqrt{\frac{2u(\alpha_0 - 1)}{(1 + \rho)\lambda c_\alpha}} (1 + u/d)^{-1} \int_0^\infty (1 + t^2)^{-1} dt \sim \frac{\pi d u^{-0.5}}{2} \sqrt{\frac{2(\alpha_0 - 1)}{(1 + \rho)\lambda c_\alpha}}. \\ \psi^{(2)}(u) &\sim \frac{\lambda}{c_\alpha} \sqrt{u} \left(1 + \frac{\sqrt{u}}{d}\right)^{-\alpha_0} \int_0^\infty \left(\frac{1}{t} + \frac{\rho\lambda\mu}{2c_\alpha} t\right)^{-\alpha_0} dt \\ &\sim \frac{\lambda}{c_\alpha} d^{\alpha_0} u^{-\frac{\alpha_0 - 1}{2}} \int_0^\infty \left(\frac{1}{t} + \frac{\rho\lambda\mu}{2c_\alpha} t\right)^{-\alpha_0} dt.\end{aligned}$$

Ruin probability for a risk process with infinite mean

In this section we are considering the following risk process.

$$R_t = R_t(u) = u + p(t) - \sum_{i=1}^{N_t} X_i$$

where the X_i are iid with distribution function F , N_t is a Poisson process with intensity λ and $p(t)$ are the premiums collected up to time t . We are interested in the infinite time ruin probability

$$\psi(u) = \mathbb{P}(\inf_{t \geq 0} R_t < 0).$$

Theorem 4.1. *If X_1, X_2, \dots are iid random variables with distribution $F(x)$ that is regularly varying with index $0 < \alpha \leq 1$, and regularly varying density $f(x)$. If further $p(T)$ is regularly varying with index $\beta > 1/\alpha$ (continuous and strictly increasing) then*

$$\psi(u) \sim \lambda \int_0^\infty \bar{F}(u + p(T)) dT \sim \lambda p^{-1}(u) \bar{F}(u) \int_0^\infty (1 + t^\beta)^{-\alpha} dt$$

Introduction

Ehrlich & Becker (1972) → 2 types of economic prevention :

- **Self-protection** : **Decrease claim probability**
 - Complements insurance
 - Observable
- **Self-insurance** : **Decrease claim amounts**
 - May substitute insurance
 - Non observable
- **Comments on model**
 - Is prevention observable?
 - 2-period models and deterministic claim amounts
 - Rational individual (expected utility)
 - Prevention demand model

Outline of the talk

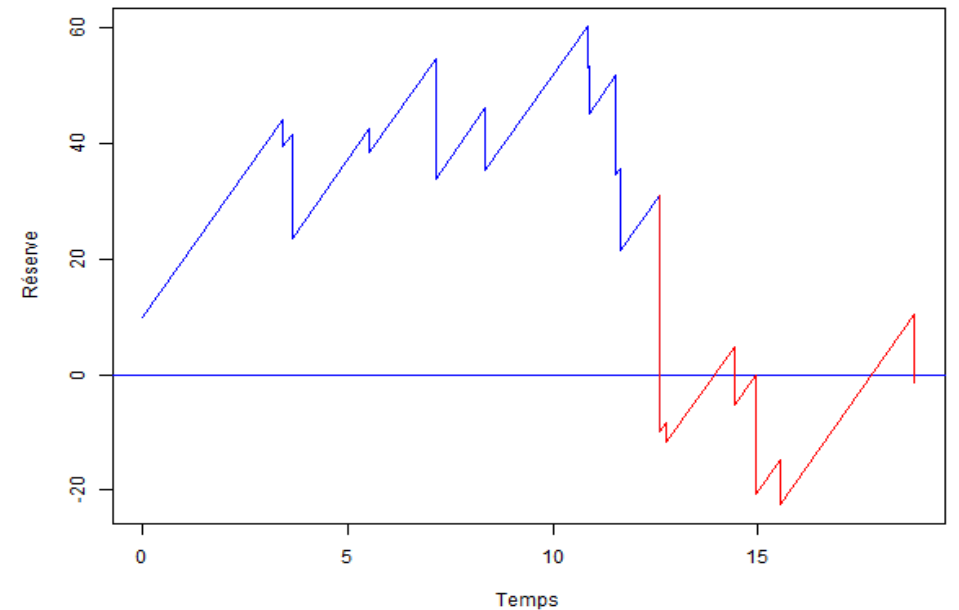
- Optimal prevention with one type of claims
- Optimal prevention with two types of claims
- How to target the right policholders in terms of prevention?

Classical notation

$$R_t = u + c t - \sum_{i=1}^{N_t} X_i$$

You all know this notation, I'm too lazy to write it down.

Non ruin probability: $\varphi(u) = \mathbb{P}(\forall t > 0, R_t > 0)$



Modeling self insurance

We assume that the prevention effort is constant over time.

- $p \in [0, c[$: prevention investment per unit of time
- h : nonincreasing function

Model :

$$R_t^{SI}(p) = u + (c - p) t - \sum_{i=1}^{N_t} h(X_i, p)$$

→ Reinsurance models

Waters, H. R. (1983). Some mathematical aspects of reinsurance. *Insurance: Mathematics and Economics*, 2(1), 17-26.

Modeling self-protection

We assume that the prevention effort is constant over time.

Soit

- $\boldsymbol{p} \in [0, \boldsymbol{c}[$: prevention investment per unit of time
- $\lambda(\cdot)$: positive, decreasing and convex
- $N_t(\boldsymbol{p})$: Poisson process with intensity $\lambda(\boldsymbol{p})$

Model:

$$R_t(\boldsymbol{p}) = \boldsymbol{u} + (\boldsymbol{c} - \boldsymbol{p}) \boldsymbol{t} - \sum_{i=1}^{N_t(\boldsymbol{p})} X_i$$

$$\varphi(\boldsymbol{u}, \boldsymbol{p}) = \mathbb{P}(\forall t > 0, R_t(\boldsymbol{p}) > 0)$$

Maximise surplus at a given time

If $\mathbf{1} \leq \frac{\lambda(p)\mu}{c-p}$, then $\forall u \geq 0, \varphi(u, p) = 0$

Problem 1 :

Find $p^{**} = \mathit{argmax} (\mathbb{E} (R_t(p)))$

Proposition 1 :

Problem 1 admits a solution iff

$$- \lambda'(0) \geq \frac{1}{\mu}.$$

Then, p^{**} satisfies

$$- \lambda'(p^{**}) = \frac{1}{\mu}.$$

Maximise non ruin probability

Problem 2 :

Find $\mathbf{p}^* = \mathit{argmax} (\varphi(\mathbf{u}, \mathbf{p}))$.

Proposition 2 :

For $u \geq 0$, Problem 2 admits a solution iff

$$-\lambda'(\mathbf{0}) - \frac{\lambda(\mathbf{0})}{c} > \mathbf{0}.$$

Then \mathbf{p}^* satisfies

$$\mathbf{p}^* = c + \frac{\lambda(\mathbf{p}^*)}{\lambda'(\mathbf{p}^*)}.$$

\mathbf{p}^* does not depend on \mathbf{u} !

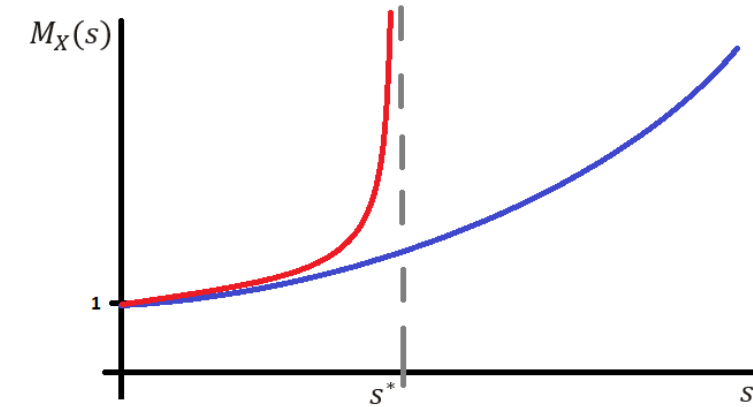
Maximise adjustment coeff.

$M_X(s)$: f.g.m. of X

Assume that $\forall s > 0, M_X(s) < \infty$,

or

$\exists s^*$ such that $\forall s \in]0, s^*[, M_X(s) < \infty$, and $\forall s \geq s^*, M_X(s) = \infty$.



Definition :

Adjustment coefficient $\kappa(p)$ such that

$$1 + \frac{c-p}{\lambda(p)} \kappa(p) = M_X(\kappa(p)).$$

Borne de Cramer Lundberg :

$$1 - \varphi(u, p) \leq e^{-\kappa(p)u}.$$

Proposition 3 :

p^* maximises κ too

Maximise dividends up to ruin

K : dividend barrier

$L_t(p)$: cumulated dividends paid up to t

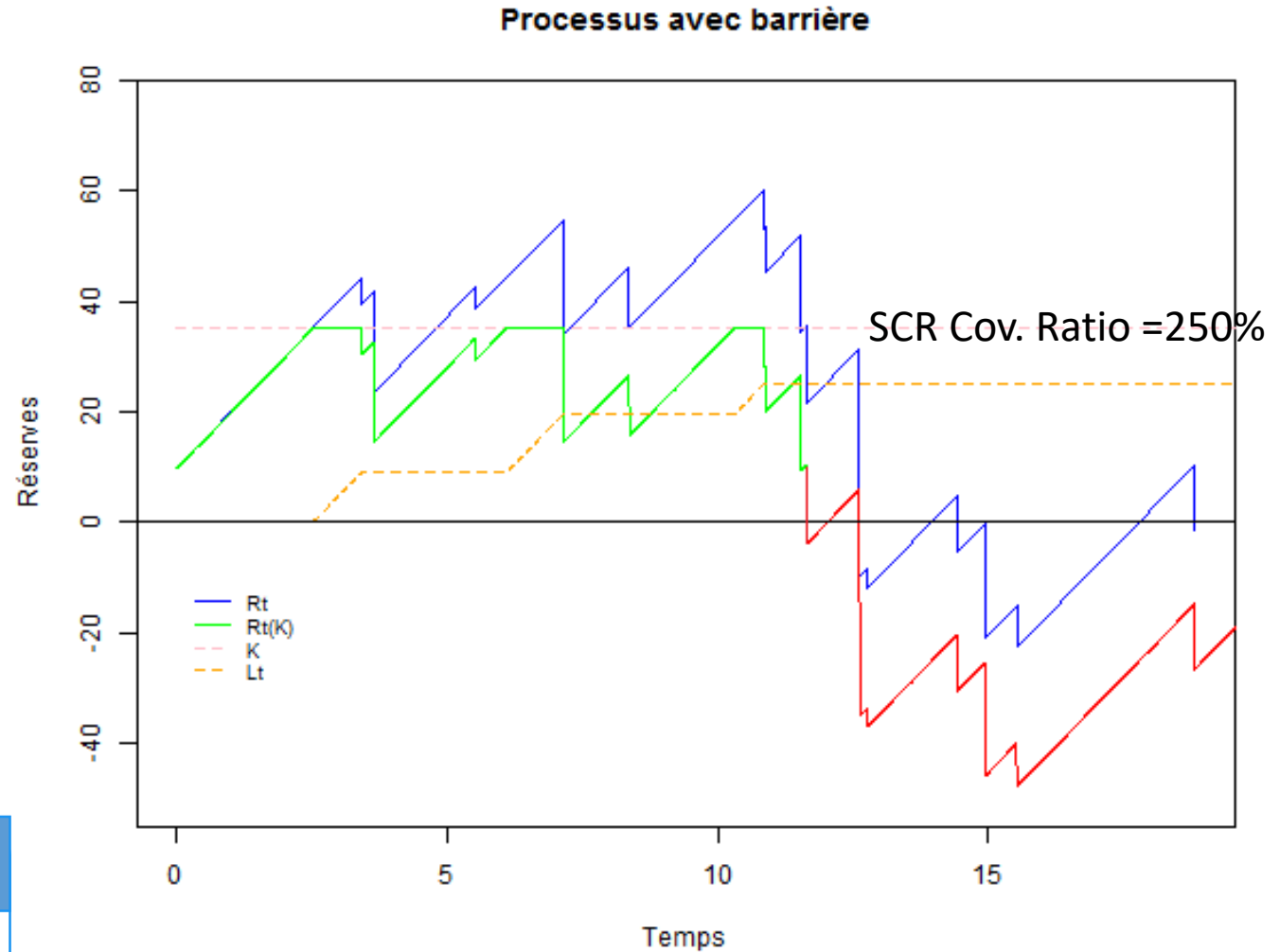
$\xi(u, K, p)$: Probability of ruin before reaching K

Segerdhal (1970) :

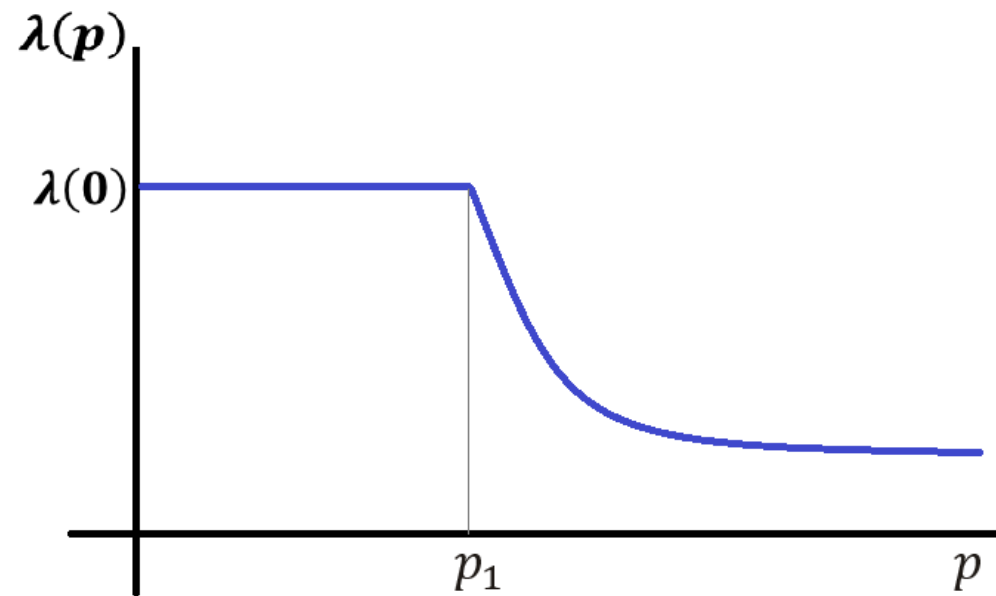
$$\mathbb{E}(L_t)(p) = \frac{(1 - \xi(u, K, p))(c - p)}{\lambda(p) \int_0^K \xi(K - x, K, p) dF_X(x)}$$

Proposition 4 :

p^* maximizes $\mathbb{E}(L_t)$ too



Extension: prevention threshold



Outline of the talk

- Optimal prevention with one type of claims
- **Optimal prevention with two types of claims**
- How to target the right policholders in terms of prevention?

Two-risk model

Two types of claims:

- **light, frequent claims** (X_i^1), average $\mu_1 < \infty$
- Poisson process N_t^1 with intensity λ_1
- **less frequent, more severe claims** (X_i^2), average $\mu_1 < \mu_2 < \infty$
- Poisson process $N_t^2(p)$ with intensity $\lambda_2(p)$

$$R_t(p) = u + (c - p)t - \sum_{i=1}^{N_t^1} X_i^1 - \sum_{j=1}^{N_t^2(p)} X_j^2$$

Equivalent to

$\widehat{N}_t(p)$ with intensity $\lambda_1 + \lambda_2(p)$

Claim amounts \widehat{X}^p equal to

- X^1 with probability $\frac{\lambda_1}{\lambda_1 + \lambda_2(p)}$
- X^2 with probability $\frac{\lambda_2(p)}{\lambda_1 + \lambda_2(p)}$

$$\widehat{R}_t(p) = u + (c - p)t - \sum_{i=1}^{\widehat{N}_t(p)} \widehat{X}_i^p$$

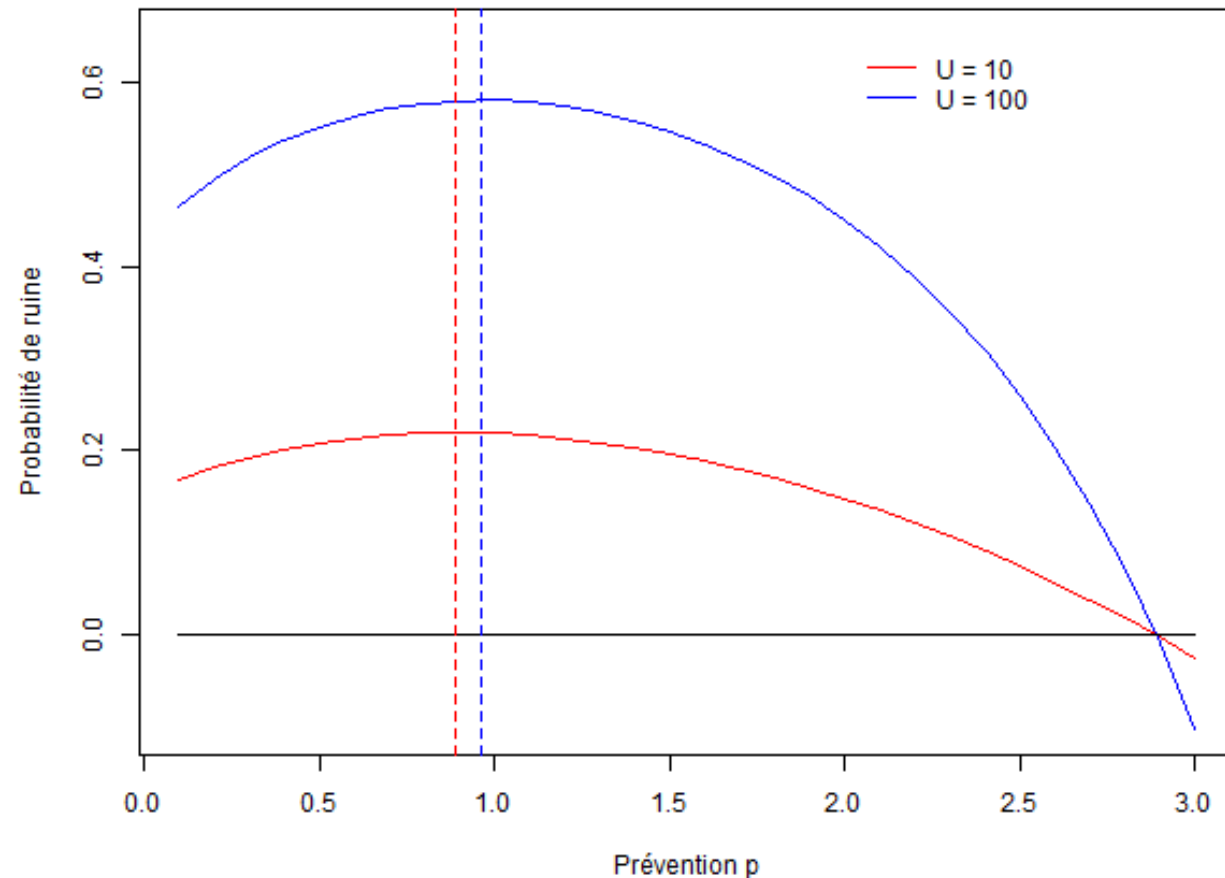
→ Self-insurance and self-prevention

Maximize non ruin probability

Problem 3

Find $p^*(u) = \operatorname{argmax} (\varphi(u, p))$

$p^*(u)$ depends on u



Proposition 7 : solution $u = 0$

For $u = 0$, Problem 3 admits a solution iff

$$-\lambda'_2(\mathbf{0}) - \frac{\lambda_1\mu_1 + \lambda_2(\mathbf{0})\mu_2}{\mu_2 c} > \mathbf{0} \quad (*)$$

Then, $p^*(0)$ satisfies

$$-\lambda'_2(p^*(0)) = \frac{\lambda_1\mu_1 + \lambda_2(p^*(0))\mu_2}{c - p^*(0)}$$

HMRL

Definition: HMRL order

$X^1 \leq_{HMRL} X^2$ if

$$\forall t \geq 0, \quad \frac{\int_t^\infty 1 - F_{X^1}(u) du}{\mathbb{E}(X^1)} \leq \frac{\int_t^\infty 1 - F_{X^2}(u) du}{\mathbb{E}(X^2)}$$

Proposition 8 : Sufficient condition for existence of $p^*(u)$

If (*) is satisfied, and if $X^1 \leq_{HMRL} X^2$, then $\forall u \geq 0$, $p^*(u)$ exists and $p^*(u) > p^*(0)$

Proposition 9 : Light-tailed asymptotics

Optimal p_κ^* satisfies

$$-\lambda'_2(p_\kappa^*) \left[(c - p_\kappa^*) + \lambda_1 \mathbb{E} \left(e^{\kappa(p_\kappa^*) \frac{\lambda_1 X^1 + \lambda_2(p_\kappa^*) X^2}{\lambda_1 + \lambda_2(p_\kappa^*)}} (X^1 - X^2) \right) \right] = \lambda_1 + \lambda_2(p_\kappa^*)$$

Besides,

$$\lim_{u \rightarrow \infty} p^*(u) = p_\kappa^*$$

Proposition 10 : When X_2 is heavy-tailed

For p_∞^* such that

$$-\lambda'_2(p_\infty^*) \left[\frac{\mu_2}{\varphi(\mathbf{0}, p_\infty^*)} + \frac{\mu_1 \lambda_1}{\lambda_2(p_\infty^*)} \right] = \frac{\mu_1 \lambda_1 + \mu_2 \lambda_2(p_\infty^*)}{\varphi(\mathbf{0}, p_\infty^*) (c - p_\infty^*)}$$

we have

$$\lim_{u \rightarrow \infty} p^*(u) = p_\infty^*$$

From prevention before claims to prevention at claims

- Climate change adaptation may be more feasible just after a claim
- Future research: investigate the benefits of prevention at claim
- Preventing so-called by-claims to occur,
- or stopping Hawkes-like mechanisms (INAR processes)
- Carbon footprint of claim management (BINAR processes)
- Work in progress with Naushad Mamode Khan and Charles Minier

Papers are available at <http://sl.isfa.fr>

THANK YOU

